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# Image Theory EM Fields Of Horizontal Dipole Antennas In Presence of Conducting Half-Space

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### **Preface**

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) It is the purpose of this report to extend the use of quasi-static range finitely conducting earth-image theory techniques to the nearfield and farfield ranges. Simple engineering expressions for horizontal electric and magnetic dipole antennas have been derived for the air-to-air, subsurface-to-air, air-to-subsurface, and subsurface-to-subsurface propagation cases. These expressions have been compared with previously derived analytical (or numerical integration) results. For the air-to-air propagation case, the image-theory expressions are valid from the quasi-static		

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to the farfield ranges, as long as the square of the index of refraction is large and the Sommerfeld numerical distance is small. For the subsurface-to-air, air-to-subsurface, and subsurface-to-subsurface propagation cases, the additional restriction that the measurement distance be greater than three times the burial depth of the source and/or receiver must be met.

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# IMAGE-THEORY ELECTROMAGNETIC FIELDS OF HORIZONTAL DIPOLE ANTENNAS IN PRESENCE OF CONDUCTING HALF-SPACE

## INTRODUCTION

During the past several years, finitely conducting earth-image theory techniques have proved quite useful in determining the quasi-static fields of antennas located near the earth's surface for both single-layered and multi-layered earths. (For detailed references, see Bannister.<sup>1,2</sup>) The quasi-static range is defined as that range where the measurement distance is much less than a free-space wavelength.

Physically, the essence of the quasi-static range finitely conducting earth-image theory technique is to replace the finitely conducting earth by a perfectly conducting earth located at the (complex) depth  $d/2$ , where  $d = 2/\gamma_1$  and  $\gamma_1 = [i\omega\mu_0(\sigma_1 + i\omega\epsilon_1)]^{1/2}$  is the propagation constant in the earth. (See figure 1 for the image-theory geometry.) Analytically, this corresponds to replacing the algebraic "reflection coefficient,"  $(u_1 - \lambda)/(u_1 + \lambda)$ , in the exact integral expressions by  $\exp(-\lambda d)$ , where  $\lambda$  is the variable of integration.<sup>3</sup> For antennas located at or above the earth's surface, the general image-theory approximation is valid throughout the quasi-static range.<sup>1,2</sup>

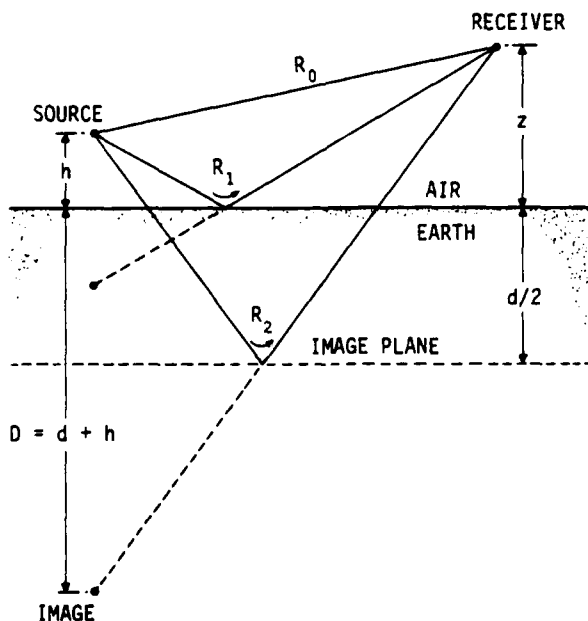


Figure 1. Image-Theory Geometry

The major disadvantage of the finitely conducting earth-image theory technique is that mainly it has been applied only in the quasi-static range. However, because this range includes the critical launching area, one would expect to be able to extend it into the intermediate range where the principal field propagation proceeds as though it were over a perfectly conducting plane.

Recently<sup>2,4</sup> we have shown, for horizontally polarized sources, that finitely conducting earth-image theory techniques are not limited to the quasi-static range alone. That is, by replacing the horizontally polarized algebraic "reflection coefficient,"  $(u_1 - u_0)/(u_1 + u_0)$ , by  $\exp(-u_0 d)$ , we demonstrated that finitely conducting earth-image theory techniques can be utilized at any range from the source. Mohsen<sup>5</sup> has validated and extended these results to include higher order terms that correspond to multiple images at the same location. Mahmoud and Metwally,<sup>6</sup> employing discrete and discrete-plus-continuous images, have computed satisfactorily the change in the input impedance of a vertical magnetic dipole (VMD) due to the presence of the earth.

It is the purpose of this report to show that nearfield and farfield range finitely conducting earth-image theory techniques also can be employed for determining the fields produced by horizontal electric dipole (HED) and horizontal magnetic dipole (HMD) antennas (which are a combination of vertically and horizontally polarized sources).

In this report, the HED and HMD are situated at height  $h$  with respect to a cylindrical coordinate system  $(\rho, \phi, z)$  and are assumed to carry a constant current  $I$ . The HED (of infinitesimal length  $\ell$ ) is oriented in the  $x$  direction while the axis of the HMD (of infinitesimal area  $A$ ) is oriented in the  $y$  direction. The earth, which is assumed to be a homogeneous medium with conductivity  $\sigma_1$  and dielectric constant  $\epsilon_1 (= \epsilon_r \epsilon_0)$ , occupies the lower half-space ( $z < 0$ ) and the air occupies the upper half-space ( $z > 0$ ). The magnetic permeability of the earth is assumed to equal  $\mu_0$ , the permeability of free space. Meter-kilogram-second (MKS) units are employed and a suppressed time factor of  $\exp(i\omega t)$  is assumed.

#### LOCATION OF IMAGE DEPTH

Basic antenna theory tells us that the fields produced by a current-carrying wire of any length, when placed over a perfectly conducting earth, can be represented by the combined fields of the wire and its image.<sup>7</sup> If the finitely conducting earth could be replaced with a perfectly conducting earth at some specified depth below the surface of the finitely conducting earth, we then could use standard image theory to locate the antenna image and the resulting fields.

Several methods are available for deriving the depth of a perfectly conducting plane that can be used to replace a finitely conducting earth. One of the most general methods<sup>8</sup> is to equate the wave impedances for grazing incidence at the surface ( $z = 0$ ) for the two cases shown in figure 2.



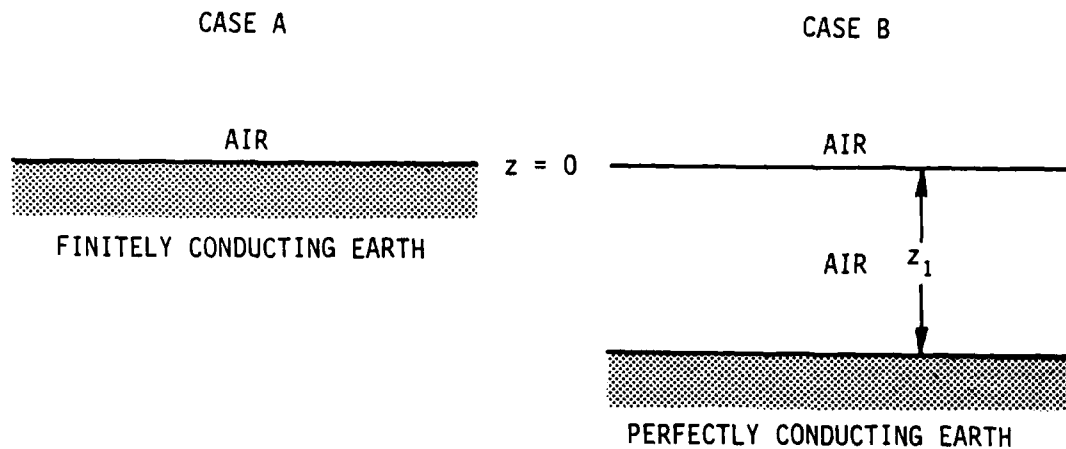


Figure 2. Replacement of a Finitely Conducting Earth With a Perfectly Conducting Earth at Depth  $z_1$

For case A, for transverse electric (TE) propagation,

$$Z_A = \frac{\eta_1}{\sqrt{1 - \gamma_0^2/\gamma_1^2}}, \quad (1)$$

where  $\gamma_0^2 = -\omega^2\mu_0\epsilon_0$ ,  $\gamma_1^2 = i\omega\mu_0(\sigma_1 + i\omega\epsilon_1)$ , and

$$\eta_1 = \left( \frac{i\omega\mu_0}{\sigma_1 + i\omega\epsilon_1} \right)^{1/2}. \quad (2)$$

For case B, we can write

$$Z_B = \eta_0 \tanh(\gamma_0 z_1), \quad (3)$$

where  $\eta_0 = \sqrt{\mu_0/\epsilon_0} \sim 120\pi$ .

For small values of  $z_1$  (i.e.,  $|\gamma_0 z_1| \leq 0.5$ ),  $\tanh(\gamma_0 z_1) \sim \gamma_0 z_1$ , and

$$Z_B = \eta_0 \gamma_0 z_1 = i\omega\mu_0 z_1. \quad (4)$$

Equating the two impedances results in

$$z_1 = \frac{1}{\gamma_1 \sqrt{1 - \gamma_0^2/\gamma_1^2}} = \frac{1}{\sqrt{\gamma_1^2 - \gamma_0^2}}. \quad (5)$$

Since the image depth is equal to  $2z_1$ , we see that, for TE propagation, the image depth  $d_{TE}$  for a wire on the surface of a finitely conducting earth can be expressed as

$$d_{TE} = \frac{2}{\sqrt{\gamma_1^2 - \gamma_0^2}} = \frac{2/\gamma_1}{\sqrt{1 - 1/n^2}}, \quad (6)$$

subject to the condition that  $|\gamma_0 z_1| \leq 0.5$  (i.e.,  $|\sqrt{n^2 - 1}| \geq 2$ , where  $n^2 = \gamma_1^2/\gamma_0^2$ ).

Similarly, for transverse magnetic (TM) propagation, since

$$z_A = \eta_1 \sqrt{1 - \gamma_0^2/\gamma_1^2}, \quad (7)$$

then,

$$d_{TM} = \frac{2}{\gamma_1} \sqrt{1 - \gamma_0^2/\gamma_1^2} = \frac{2}{\gamma_1} \sqrt{1 - 1/n^2}, \quad (8)$$

subject to the condition that  $|\gamma_0 z_1| \leq 0.5$  (i.e.,  $|n^2/\sqrt{n^2 - 1}| \geq 2$ ).

For normal incidence, or if  $|n^2| > 15$ , equations (6) and (8) reduce to the well-known result<sup>1,2,3,8</sup>

$$d_{TE} = d_{TM} = d = 2/\gamma_1, \quad (9)$$

where  $\gamma_1 = \sqrt{i\omega\mu_0\sigma_1}$  for  $\sigma_1 \gg \omega\epsilon_0\epsilon_r$  and  $\gamma_1 = 60\pi\sigma_1/\sqrt{\epsilon_r} + i\omega\sqrt{\mu_0\epsilon_0\epsilon_r}$  for  $\sigma_1 \ll \omega\epsilon_0\epsilon_r$ .

Another way to determine the finitely conducting earth-image depth is to compare the results obtained from image theory with known analytical results. For  $z = h = 0$ , the HED Hertz vector is exactly equal to<sup>9</sup>

$$\Pi_x = \frac{I\ell}{4\pi i\omega\epsilon_0} \times \frac{2}{(\gamma_1^2 - \gamma_0^2)\rho^3} \left[ (1 + \gamma_0\rho)e^{-\gamma_0\rho} - (1 + \gamma_1\rho)e^{-\gamma_1\rho} \right]. \quad (10)$$

The image-theory result is

$$\Pi_x = \frac{I\ell}{4\pi i\omega\epsilon_0} \left[ \frac{e^{-\gamma_0\rho}}{\rho} - \frac{e^{-\gamma_0\rho_i}}{\rho_i} \right], \quad (11)$$

where

$$\rho_i = (\rho^2 + d_{TE}^2)^{1/2}.$$

When  $\text{Re}\gamma_1\rho \gg 1$ , equation (10) reduces to

$$\Pi_x = \frac{I l}{4\pi i \omega \epsilon_0 \rho^3} \left( \frac{2}{\gamma_1^2 - \gamma_0^2} \right) (1 + \gamma_0 \rho) e^{-\gamma_0 \rho}, \quad (12)$$

while equation (11) becomes

$$\Pi_x = \frac{I l}{4\pi i \omega \epsilon_0 \rho^3} \left( \frac{d_{TE}^2}{2} \right) (1 + \gamma_0 \rho) e^{-\gamma_0 \rho}. \quad (13)$$

Equating equations (12) and (13) results in

$$d_{TE} = \frac{2}{\sqrt{\gamma_1^2 - \gamma_0^2}}, \quad (14)$$

which is identical to equation (6).

Following Wait and Spies,<sup>3</sup> another way to determine the image depth is to expand the function

$$f(u_0) = e^{u_0 d} \left( \frac{u_1 - u_0}{u_1 + u_0} \right) \quad (15)$$

in a Taylor series about  $u_0 = 0$ , resulting in

$$\frac{u_1 - u_0}{u_1 + u_0} = e^{-u_0 d} \left[ 1 + \frac{1}{3} \left( \frac{u_0 d}{2} \right)^3 + \dots \right], \quad (16)$$

where  $d$  is given by equation (6).

The introduction of  $\exp(-u_0 d)$  into the  $\Pi_x$  integral equation yields an image at a distance  $h + d$  from the earth's surface (see figure 1), while higher order terms would correspond to multiple images at the same location.<sup>5,6</sup>

#### HED AIR-TO-AIR PROPAGATION

When  $h$  and  $z$  are  $\geq 0$ , the Sommerfeld integral expressions for the HED Hertz vector are<sup>9-11</sup>

$$\Pi_x = \frac{I l}{4\pi i \omega \epsilon_0} \left\{ \frac{e^{-\gamma_0 R_0}}{R_0} - \frac{e^{-\gamma_0 R_1}}{R_1} + \int_0^\infty \left( \frac{2u_0}{u_1 + u_0} \right) e^{-u_0(z+h)} J_0(\lambda \rho) \frac{\lambda}{u_0} d\lambda \right\} \quad (17)$$

and

$$\Pi_z = \frac{I l \cos \phi}{4\pi i \omega \epsilon_0} \times \frac{\partial}{\partial \rho} \int_0^\infty \frac{2(u_1 - u_0)}{\gamma_1^2 u_0 + \gamma_0^2 u_1} e^{-u_0(z+h)} J_0(\lambda \rho) \lambda d\lambda, \quad (18)$$

where  $R_0^2 = \rho^2 + (z - h)^2$ ,  $R_1^2 = \rho^2 + (z + h)^2$ ,  $u_0^2 = \lambda^2 + \gamma_0^2$ , and  $u_1^2 = \lambda^2 + \gamma_1^2$ .

From equations (17) and (18),

$$\vec{\nabla} \cdot \vec{\Pi} = \frac{I l \cos \phi}{4\pi i \omega \epsilon_0} \times \frac{\partial}{\partial \rho} \left\{ \frac{e^{-\gamma_0 R_0}}{R_0} - \frac{e^{-\gamma_0 R_1}}{R_1} + \int_0^\infty \frac{2\gamma_0^2 e^{-u_0(z+h)}}{\gamma_1^2 u_0 + \gamma_0^2 u_1} J_0(\lambda \rho) \lambda d\lambda \right\}. \quad (19)$$

If  $|\gamma_0 z_1| \leq 0.5$  (i.e.,  $|\sqrt{n^2 - 1}| \geq 2$ ), which is applicable in most practical cases (for example,  $|n^2| \geq 81$  at all frequencies for the sea/air case),

$$\frac{u_1 - u_0}{u_1 + u_0} \sim e^{-u_0 d} \quad (20)$$

and

$$1 - \left( \frac{u_1 - u_0}{u_1 + u_0} \right) = \frac{2u_0}{u_1 + u_0} \sim 1 - e^{-u_0 d}. \quad (21)$$

For small Sommerfeld numerical distances (i.e.,  $|\gamma_0 R_1 \sqrt{n^2 - 1}/(2n^3)| \ll 1$ ),

$$\gamma_1^2 u_0 + \gamma_0^2 u_1 \sim \gamma_1^2 u_0. \quad (22)$$

Utilization of the identity  $(u_1 - u_0)(u_1 + u_0) = \gamma_1^2 - \gamma_0^2$ , equations (21) and (22), and Sommerfeld's integral<sup>10</sup>

$$P = \int_0^\infty e^{-u_0(z+h)} J_0(\lambda \rho) \frac{\lambda}{u_0} d\lambda = \frac{e^{-\gamma_0 R_1}}{R_1}, \quad (23)$$

results in

$$\begin{aligned} \Pi_x &= \frac{I l}{4\pi i \omega \epsilon_0} \left\{ \frac{e^{-\gamma_0 R_0}}{R_0} - \frac{e^{-\gamma_0 R_1}}{R_1} + \frac{e^{-\gamma_0 R_1}}{R_1} - \frac{e^{-\gamma_0 R_2}}{R_2} \right\} \\ &\quad - \frac{I l}{4\pi i \omega \epsilon_0} \left\{ \frac{e^{-\gamma_0 R_0}}{R_0} - \frac{e^{-\gamma_0 R_2}}{R_2} \right\} \end{aligned} \quad (24)$$

$$\begin{aligned} \vec{\nabla} \cdot \vec{\Pi} &= \frac{I l \cos \phi}{4\pi i \omega \epsilon_0} \times \frac{\partial}{\partial \rho} \left\{ \frac{e^{-\gamma_0 R_0}}{R_0} - \left( 1 - \frac{2}{n^2} \right) \frac{e^{-\gamma_0 R_1}}{R_1} \right\} \\ &= - \frac{I l \rho \cos \phi}{4\pi i \omega \epsilon_0} \left\{ (1 + \gamma_0 R_0) \frac{e^{-\gamma_0 R_0}}{R_0^3} - \left( 1 - \frac{2}{n^2} \right) (1 + \gamma_0 R_1) \frac{e^{-\gamma_0 R_1}}{R_1^3} \right\} \end{aligned} \quad (25)$$

and

$$\begin{aligned} \Pi_z &= \frac{I l \cos \phi}{4\pi i \omega \epsilon_0} (1 - 1/n^2) \frac{\partial}{\partial \rho} \int_0^\infty \left(1 - e^{-u_0 d}\right) e^{-u_0(z+h)} J_0(\lambda \rho) \frac{\lambda}{u_0^2} d\lambda \\ &= - \frac{I l \cos \phi}{4\pi i \omega \epsilon_0} (1 - 1/n^2) \int_0^\infty \left(1 - e^{-u_0 d}\right) e^{-u_0(z+h)} J_1(\lambda \rho) \frac{\lambda^2}{u_0^2} d\lambda, \end{aligned} \quad (26)$$

where  $R_2^2 = \rho^2 + (d + z + h)^2$ .

Because of the  $u_0^2$  term in the denominator of equation (26), it does not readily appear that the equation can be expressed in closed form, except for the quasi-static range ( $\gamma_0 \sim 0$ ), where equation (26) reduces to

$$\begin{aligned} \Pi_z &= - \frac{I l \cos \phi}{4\pi i \omega \epsilon_0} \int_0^\infty \left(1 - e^{-\lambda d}\right) e^{-\lambda(z+h)} J_1(\lambda \rho) d\lambda \\ &= - \frac{I l \cos \phi}{4\pi i \omega \epsilon_0 \rho} \left[ \frac{(d + z + h)}{R_2} - \frac{(z + h)}{R_1} \right], \end{aligned} \quad (27)$$

However, if we replace  $\lambda^2$  by  $u_0^2 - \gamma_0^2$ , equation (26) can be broken up into four integrals, two of which are of the type

$$\int_0^\infty e^{-u_0 z} J_1(\lambda \rho) d\lambda = \frac{1}{\rho} \left[ e^{-\gamma_0 z} - \frac{z}{R} e^{-\gamma_0 R} \right], \quad (28)$$

where  $R^2 = \rho^2 + z^2$ , and two of which are of the type

$$\gamma_0^2 \int_0^\infty \frac{e^{-u_0 z}}{u_0^2} J_1(\lambda \rho) d\lambda. \quad (29)$$

Since<sup>12</sup>

$$\int_0^\infty \frac{e^{-u_0 z}}{u_0} J_1(\lambda \rho) d\lambda = \frac{1}{\gamma_0 \rho} \left[ e^{-\gamma_0 z} - e^{-\gamma_0 R} \right], \quad (30)$$

then

$$\gamma_0^2 \int_0^\infty \frac{e^{-u_0 z}}{u_0^2} J_1(\lambda \rho) d\lambda = \frac{e^{-\gamma_0 z}}{\rho} + \frac{\gamma_0}{\rho} \int_z^\infty e^{-\gamma_0 R} dz. \quad (31)$$

Inserting equations (28) and (31) into equation (26) results in

$$\Pi_z \sim - \frac{I \ell \cos \phi (1 - 1/n^2)}{4\pi i \omega \epsilon_0 \rho} \left\{ \frac{(d + z + h)}{R_2} e^{-\gamma_0 R_2} - \frac{(z + h)}{R_1} e^{-\gamma_0 R_1} + I \right\}, \quad (32)$$

where

$$I = -\gamma_0 \int_z^\infty e^{-\gamma_0 R_1} \left[ 1 - e^{-\gamma_0 (R_2 - R_1)} \right] dz. \quad (33)$$

When  $(z + h) \gg \rho$ ,  $R_1 \sim (z + h)$ , and  $R_2 - R_1 \sim d$ , resulting in

$$I \sim \left( 1 - e^{-\gamma_0 d} \right) e^{-\gamma_0 (z+h)}. \quad (34)$$

For  $|\gamma_0 d| < 0.5$  (i.e.,  $|n^2| > 15$ ),

$$I \sim \gamma_0 d e^{-\gamma_0 (z+h)}. \quad (35)$$

Comparing equations (27) and (32), we see that for the quasi-static range ( $|\gamma_0 R_1| \ll 1$ ),  $I$  is negligible compared to the other two terms in equation (32). Therefore, if we assume from the outset that  $R_1 \gg |d|$ , equation (33) becomes

$$I \sim \gamma_0 d e^{-\gamma_0 R_1}, \quad (36)$$

which is identical to equation (35) when  $(z + h) \gg \rho$ .

Therefore, we can approximate the HED  $\Pi_z$  vector as

$$\begin{aligned} \Pi_z \sim - \frac{I \ell \cos \phi (1 - 1/n^2)}{4\pi i \omega \epsilon_0 \rho} & \left\{ \frac{(d + z + h) e^{-\gamma_0 R_2}}{R_2} \right. \\ & \left. - \frac{(z + h) e^{-\gamma_0 R_1}}{R_1} + \gamma_0 d e^{-\gamma_0 R_1} \right\}, \end{aligned} \quad (37)$$

subject to the restriction  $|n^2| > 15$ . This equation should also be valid for  $|n^2| > 5$  if  $\gamma_0 d$  is replaced by  $(1 - e^{-\gamma_0 d})$ .

Now, it seems that we have gone to a lot of trouble to derive the last term of the  $\Pi_z$  expression. However, it is this term that yields the vertically polarized farfield (for small numerical distances) produced by a HED located

very near or below the earth's surface. This is the so-called quadripole term (i.e., the quadripole moment is  $12\gamma_0 d = 212/n$ ). Because of the close spacing and opposite sense of the dipole and its image, direct radiation from the dipole is not the prime mechanism. Rather, the quadripole consisting of vertical conduction currents in the lossy medium represents the prime source.<sup>13</sup>

If the first two terms in equation (37) are ignored and  $z = h = 0$ , equation (37) reduces to

$$\pi_z \sim - \frac{12 \cos \phi}{4\pi i \omega \epsilon_0 \rho} \left( \gamma_0 d e^{-\gamma_0 \rho} \right). \quad (38)$$

When  $|\gamma_0 \rho| \gg 1$ ,

$$H_\phi \sim - i \omega \epsilon_0 \frac{\partial \pi_z}{\partial \rho} \sim - \frac{12 \cos \phi}{2\pi \gamma_1 \rho} (\gamma_0^2) e^{-\gamma_0 \rho}, \quad (39)$$

which is the correct farfield result for small numerical distances when  $|n^2| \gg 1$ .

Since we have now derived expressions for the HED Hertz vector (equations (24), (25), and (37)), the fields in air can be obtained from

$$\begin{aligned} \vec{E} &= -\gamma_0^2 \vec{\Pi} + \vec{\nabla}(\vec{\nabla} \cdot \vec{\Pi}) \\ H &= i \omega \epsilon_0 (\vec{\nabla} \times \vec{\Pi}). \end{aligned} \quad (40)$$

The resulting HED finitely conducting earth-image theory field expressions for the air-to-air propagation case are presented in table 1. They are valid for small numerical distances and  $|n^2| > 15$ . When  $|\gamma_0 R_1| \ll 1$ , they reduce to the quasi-static range image-theory results.<sup>1,2</sup> When  $|\gamma_0 R_1| \gg 1$ , they reduce to Norton's<sup>14,15</sup> farfield results.

These results easily can be extended to a multilayered earth simply by letting  $d = (2/\gamma_1)Q$ , where  $Q$  is the familiar plane-wave correction factor employed to account for the presence of stratification in the earth.<sup>16,17</sup>

#### HMD AIR-TO-AIR PROPAGATION

Four of the six HMD expressions valid for  $h$  and  $z \geq 0$  can be obtained completely from reciprocity consideration ( $E_\rho$ ,  $E_\phi$ ,  $E_z$ , and  $H_z$ ). The remaining two can be obtained from Maxwell's equations ( $H_\rho$  and  $H_\phi$ ). Alternatively, they can be obtained from

Table 1. HED Air-to-Air Propagation Equations.

$$\begin{aligned}
E_{\rho} &= \frac{I l \cos \phi}{4 \pi i \omega \epsilon_0} \left\{ \left[ \left( \frac{3 \rho^2}{R_0^2} - 1 \right) (1 + \gamma_0 R_0) + \gamma_0^2 \rho^2 \right] \frac{e^{-\gamma_0 R_0}}{R_0^3} - \left[ 1 - \frac{2}{n^2} \right] \left[ \left( \frac{3 \rho^2}{R_1^2} - 1 \right) \right. \right. \\
&\quad \left. \left. \cdot (1 + \gamma_0 R_1) + \gamma_0^2 \rho^2 \right] \frac{e^{-\gamma_0 R_1}}{R_1^3} - \gamma_0^2 \left[ \frac{e^{-\gamma_0 R_0}}{R_0} - \frac{e^{-\gamma_0 R_2}}{R_2} \right] \right\} \\
E_{\phi} &= \frac{I l \sin \phi}{4 \pi i \omega \epsilon_0} \left\{ (1 + \gamma_0 R_0) \frac{e^{-\gamma_0 R_0}}{R_0^3} - \left( 1 - \frac{2}{n^2} \right) (1 + \gamma_0 R_1) \frac{e^{-\gamma_0 R_1}}{R_1^3} \right. \\
&\quad \left. + \gamma_0^2 \left( \frac{e^{-\gamma_0 R_0}}{R_0} - \frac{e^{-\gamma_0 R_2}}{R_2} \right) \right\} \\
E_z &= \frac{I l \rho \cos \phi}{4 \pi i \omega \epsilon_0} \left\{ \frac{(z - h)}{R_0^5} (3 + 3 \gamma_0 R_0 + \gamma_0^2 R_0^2) e^{-\gamma_0 R_0} \right. \\
&\quad - \left( 1 - \frac{2}{n^2} \right) \frac{(z + h)}{R_1^5} (3 + 3 \gamma_0 R_1 + \gamma_0^2 R_1^2) e^{-\gamma_0 R_1} \\
&\quad \left. + \frac{\gamma_0^2}{\rho^2} \left[ \frac{(d + z + h)}{R_2} e^{-\gamma_0 R_2} + \frac{\gamma_0 d R_1 - (z + h)}{R_1} e^{-\gamma_0 R_1} \right] \right\} \\
H_{\rho} &= \frac{I l \sin \phi}{4 \pi} \left\{ \frac{(d + z + h)}{R_2} \left[ \frac{1}{\rho^2} + \frac{1}{R_2^2} (1 + \gamma_0 R_2) \right] e^{-\gamma_0 R_2} \right. \\
&\quad \left. + \left[ \frac{\gamma_0 d R_1 - (z + h)}{\rho^2 R_1} \right] e^{-\gamma_0 R_1} - \frac{(z - h)}{R_0^3} (1 + \gamma_0 R_0) e^{-\gamma_0 R_0} \right\} \\
H_{\phi} &= - \frac{I l \cos \phi}{4 \pi} \left\{ \frac{(z - h)}{R_0^3} (1 + \gamma_0 R_0) e^{-\gamma_0 R_0} - \frac{(z + h)}{R_1^3} (1 + \gamma_0 R_1) e^{-\gamma_0 R_1} \right. \\
&\quad \left. + \frac{1}{\rho^2} \left[ \frac{(d + z + h)}{R_2} e^{-\gamma_0 R_2} - \frac{(z + h)}{R_1} e^{-\gamma_0 R_1} \right] + \frac{d e^{-\gamma_0 R_1}}{\rho^2 R_1} \left[ \gamma_0 R_1 + \gamma_0^2 \rho^2 \right] \right\} \\
H_z &= \frac{I l \rho \sin \phi}{4 \pi} \left\{ (1 + \gamma_0 R_0) \frac{e^{-\gamma_0 R_0}}{R_0^3} - (1 + \gamma_0 R_2) \frac{e^{-\gamma_0 R_2}}{R_2^3} \right\}
\end{aligned}$$



$$\begin{aligned}
 H_\rho &= -\gamma_0^2 \pi_y \sin \phi + \frac{\partial}{\partial \rho} (\vec{\nabla} \cdot \vec{\Pi}) \\
 H_\phi &= -\gamma_0^2 \pi_y \cos \phi + \frac{1}{\rho} \times \frac{\partial}{\partial \phi} (\vec{\nabla} \cdot \vec{\Pi}) .
 \end{aligned}
 \tag{41}$$

Following the same procedure outlined in the derivation of the HED field components results in

$$\pi_y = \frac{IA}{4\pi} \left[ \frac{e^{-\gamma_0 R_0}}{R_0} + \frac{e^{-\gamma_0 R_1}}{R_1} \right]
 \tag{42}$$

and

$$\vec{\nabla} \cdot \vec{\Pi} = - \frac{IA\rho \sin \phi}{4\pi} \left[ \frac{e^{-\gamma_0 R_0}}{R_0^3} (1 + \gamma_0 R_0) + \frac{e^{-\gamma_0 R_2}}{R_2^3} (1 + \gamma_0 R_2) \right] .
 \tag{43}$$

The resulting HMD finitely conducting earth-image theory field expressions for the air-to-air propagation case are presented in table 2. They are valid for  $|n^2| > 15$  and small numerical distances. When  $|\gamma_0 R_1| \ll 1$ , they reduce to the quasi-static range image-theory results.<sup>1,2</sup> When  $|\gamma_0 R_1| \gg 1$ , they reduce to Norton's<sup>14,15</sup> farfield results.

#### SUBSURFACE-TO-AIR PROPAGATION

The HED and HMD image-theory expressions for the subsurface-to-air propagation case ( $h \leq 0$ ,  $z \geq 0$ ) can be obtained from the air-to-air propagation equations (tables 1 and 2) simply by setting  $h = 0$  and multiplying each expression by  $\exp(\gamma_1 h)$ . The resulting equations are presented in tables 3 and 4 and will be valid<sup>18</sup> for  $R = \sqrt{\rho^2 + z^2} > |3h|$ ,  $|n^2| > 15$ , and small numerical distances. It should be noted that by following the procedure outlined by Bannister and Dube,<sup>18</sup> the restriction  $R > |3h|$  can become less stringent.

When  $|\gamma_1 R| \gg 1$ , the HED and HMD subsurface-to-air propagation equations reduce to the nearfield and farfield range results presented in tables 3.1 and 3.3 of Kraichman<sup>19</sup> (see also Bannister<sup>2</sup>).

#### AIR-TO-SUBSURFACE PROPAGATION

The HED and HMD image-theory expressions for the air-to-subsurface propagation case ( $h \geq 0$ ,  $z \leq 0$ ) can be obtained from the air-to-air propagation equations (tables 1 and 2) simply by setting  $z = 0$  and multiplying each expression by  $\exp(\gamma_1 z)$ . (Both  $E_z$  components must also be multiplied by  $1/n^2$  to satisfy the boundary conditions.) The resulting equations are presented in tables 5 and 6 and will be valid<sup>18</sup> for  $R' = \sqrt{\rho^2 + h^2} > |3z|$ ,  $|n^2| > 15$ , and small numerical distances.

Table 2. HMD Air-to-Air Propagation Equations

$$\begin{aligned}
E_{\rho} &= -\frac{i\omega\mu_0 IA \cos \phi}{4\pi} \left\{ \frac{(z-h)}{R_0^3} (1 + \gamma_0 R_0) e^{-\gamma_0 R_0} + \frac{(z+h)}{R_1^3} (1 + \gamma_0 R_1) e^{-\gamma_0 R_1} \right. \\
&\quad \left. - \frac{1}{\rho^2} \left[ \frac{(d+z+h)}{R_2} e^{-\gamma_0 R_2} - \frac{(z+h)}{R_1} e^{-\gamma_0 R_1} \right] - \frac{de^{-\gamma_0 R_1}}{\rho^2 R_1} (\gamma_0 R_1 + \gamma_0^2 \rho^2) \right\} \\
E_{\phi} &= \frac{i\omega\mu_0 IA \sin \phi}{4\pi} \left\{ \frac{(d+z+h)}{R_2} \left[ \frac{1}{\rho^2} + \frac{1}{R_2^2} (1 + \gamma_0 R_2) \right] e^{-\gamma_0 R_2} \right. \\
&\quad \left. + \left[ \frac{\gamma_0 d R_1 - (z+h)}{\rho^2 R_1} \right] e^{-\gamma_0 R_1} + \frac{(z-h)}{R_0^3} (1 + \gamma_0 R_0) e^{-\gamma_0 R_0} \right\} \\
E_z &= \frac{i\omega\mu_0 I \rho \cos \phi}{4\pi} \left\{ (1 + \gamma_0 R_0) \frac{e^{-\gamma_0 R_0}}{R_0^3} + (1 + \gamma_0 R_1) \frac{e^{-\gamma_0 R_1}}{R_1^3} \right\} \\
H_{\rho} &= \frac{IA \sin \phi}{4\pi} \left\{ \left[ \left[ 2 - \frac{3(z-h)^2}{R_0^2} \right] [1 + \gamma_0 R_0] - \gamma_0^2 (z-h)^2 \right] \frac{e^{-\gamma_0 R_0}}{R_0^3} \right. \\
&\quad \left. + \left[ \left[ 2 - \frac{3(d+z+h)^2}{R_2^2} \right] [1 + \gamma_0 R_2] + \gamma_0^2 \rho^2 \right] \frac{e^{-\gamma_0 R_2}}{R_2^3} - \frac{\gamma_0^2 R_1^2 e^{-\gamma_0 R_1}}{R_1^3} \right\} \\
H_{\phi} &= -\frac{IA \cos \phi}{4\pi} \left\{ (1 + \gamma_0 R_0 + \gamma_0^2 R_0^2) \frac{e^{-\gamma_0 R_0}}{R_0^3} \right. \\
&\quad \left. + (1 + \gamma_0 R_2) \frac{e^{-\gamma_0 R_2}}{R_2^3} + \gamma_0^2 R_1^2 \frac{e^{-\gamma_0 R_1}}{R_1^3} \right\} \\
H_z &= \frac{I \rho \sin \phi}{4\pi} \left\{ \frac{(d+z+h) e^{-\gamma_0 R_2}}{R_2^5} (3 + 3\gamma_0 R_2 + \gamma_0^2 R_2^2) \right. \\
&\quad \left. + \frac{(z-h) e^{-\gamma_0 R_0}}{R_0^5} (3 + 3\gamma_0 R_0 + \gamma_0^2 R_0^2) \right\}
\end{aligned}$$

Table 3. HED Subsurface-to-Air Propagation Equations  
 $[R^2 = \rho^2 + z^2, R_i^2 = \rho^2 + (d + z)^2]$ .

$$E_\rho = \frac{I l \cos \phi e^{-(\gamma_0 R - \gamma_1 h)}}{2\pi(\sigma_1 + i\omega\epsilon_1)R^3} \left\{ \left( \frac{3\rho^2}{R^2} - 1 \right) (1 + \gamma_0 R) + \gamma_0^2 \rho^2 - \frac{2R^2}{d^2} \left[ 1 - \frac{R}{R_i} e^{-\gamma_0(R_i - R)} \right] \right\}$$

$$E_\phi = \frac{I l \sin \phi e^{-(\gamma_0 R - \gamma_1 h)}}{2\pi(\sigma_1 + i\omega\epsilon_1)R^3} \left\{ (1 + \gamma_0 R) + \frac{2R^2}{d^2} \left[ 1 - \frac{R}{R_i} e^{-\gamma_0(R_i - R)} \right] \right\}$$

$$E_z = \frac{I l i \omega \mu_0 \cos \phi e^{-(\gamma_0 R - \gamma_1 h)}}{4\pi \rho R} \left\{ \gamma_0 R d - z + \frac{R}{R_i} (d + z) e^{-\gamma_0(R_i - R)} + \frac{\rho^2 d^2 z}{2R^4} (3 + 3\gamma_0 R + \gamma_0^2 R^2) \right\}$$

$$H_\rho = \frac{I l \sin \phi e^{-(\gamma_0 R - \gamma_1 h)}}{4\pi \rho^2 R} \left\{ \gamma_0 R d - z \left[ 1 + \frac{\rho^2}{R^2} (1 + \gamma_0 R) \right] + (d + z) \frac{R}{R_i} e^{-\gamma_0(R_i - R)} \left[ 1 + \frac{\rho^2}{R_i^2} (1 + \gamma_0 R_i) \right] \right\}$$

$$H_\phi = -\frac{I l \cos \phi e^{-(\gamma_0 R - \gamma_1 h)}}{4\pi \rho^2 R} \left\{ d(\gamma_0 R + \gamma_0^2 \rho^2) - z + \frac{R}{R_i} (d + z) e^{-\gamma_0(R_i - R)} \right\}$$

$$H_z = \frac{I l \rho \sin \phi e^{-(\gamma_0 R - \gamma_1 h)}}{4\pi R^3} \left\{ (1 + \gamma_0 R) - \frac{R^3}{R_i^3} (1 + \gamma_0 R_i) e^{-\gamma_0(R_i - R)} \right\}$$

Table 4. HMD Subsurface-to-Air Propagation Equations  
 $[R^2 = \rho^2 + z^2, R_i^2 = \rho^2 + (d + z)^2]$

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$$E_\rho = \frac{i\omega\mu_0 IA \cos \phi}{4\pi\rho^2 R} e^{-(\gamma_0 R - \gamma_1 h)} \left\{ d(\gamma_0 R + \gamma_0^2 \rho^2) - \frac{2z\rho^2}{R^2}(1 + \gamma_0 R) + (d + z)\frac{R}{R_i} e^{-\gamma_0(R_i - R)} - z \right\}$$

$$E_\phi = \frac{i\omega\mu_0 IA \sin \phi}{4\pi\rho^2 R} e^{-(\gamma_0 R - \gamma_1 h)} \left\{ d\gamma_0 R - z + \frac{\rho^2 z}{R^2}(1 + \gamma_0 R) + (d + z)\frac{R}{R_i} \left[ 1 + \frac{\rho^2}{R_i^2}(1 + \gamma_0 R_i) \right] e^{-\gamma_0(R_i - R)} \right\}$$

$$E_z = \frac{i\omega\mu_0 IA \rho \cos \phi}{2\pi R^3} (1 + \gamma_0 R) e^{-(\gamma_0 R - \gamma_1 h)}$$

$$H_\rho = \frac{IA \sin \phi}{4\pi R^3} e^{-(\gamma_0 R - \gamma_1 h)} \left\{ \left( 2 - \frac{3z^2}{R^2} \right) (1 + \gamma_0 R) - 2\gamma_0^2 z^2 - \gamma_0^2 \rho^2 + \left( \frac{R}{R_i} \right)^3 e^{-\gamma_0(R_i - R)} \left[ \left( 2 - \frac{3(d + z)^2}{R_i^2} \right) [1 + \gamma_0 R_i] + \gamma_0^2 \rho^2 \right] \right\}$$

$$H_\phi = -\frac{IA \cos \phi}{4\pi R^3} e^{-(\gamma_0 R - \gamma_1 h)} \left\{ 1 + \gamma_0 R + 2\gamma_0^2 R^2 + \left( \frac{R}{R_i} \right)^3 (1 + \gamma_0 R_i) e^{-\gamma_0(R_i - R)} \right\}$$

$$H_z = \frac{IA \rho \sin \phi}{4\pi R^5} e^{-(\gamma_0 R - \gamma_1 h)} \left\{ z(3 + 3\gamma_0 R + \gamma_0^2 R^2) + (d + z) \left( \frac{R}{R_i} \right)^5 (3 + 3\gamma_0 R_i + \gamma_0^2 R_i^2) e^{-\gamma_0(R_i - R)} \right\}$$


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Table 5. HED Air-to-Subsurface Propagation Equations  
 $[(R')^2 = \rho^2 + h^2, (R'_1)^2 = \rho^2 + (d + h)^2]$

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$$E_\rho = \frac{I l \cos \phi e^{-(\gamma_0 R' - \gamma_1 z)}}{2\pi(\sigma_1 + i\omega\epsilon_1)(R')^3} \left\{ \left[ \frac{3\rho^2}{(R')^2} - 1 \right] [1 + \gamma_0 R'] \right.$$

$$\left. + \gamma_0^2 \rho^2 - \frac{2(R')^2}{d^2} \left[ 1 - \frac{R'_1}{R'_i} e^{-\gamma_0(R'_1 - R')} \right] \right\}$$

$$E_\phi = \frac{I l \sin \phi e^{-(\gamma_0 R' - \gamma_1 z)}}{2\pi(\sigma_1 + i\omega\epsilon_1)(R')^3} \left\{ 1 + \gamma_0 R' + \frac{2(R')^2}{d^2} \left[ 1 - \frac{R'_1}{R'_i} e^{-\gamma_0(R'_1 - R')} \right] \right\}$$

$$E_z = - \frac{I l \rho \cos \phi e^{-(\gamma_0 R' - \gamma_1 z)}}{2\pi(\sigma_1 + i\omega\epsilon_1)(R')^5} \left\{ h [3 + 3\gamma_0 R' + \gamma_0^2 (R')^2] \right.$$

$$\left. - \frac{\gamma_0^2 (R')^4}{2\rho^2} \left[ \gamma_0 d R' - h + \frac{R'_1}{R'_i} (d + h) e^{-\gamma_0(R'_1 - R')} \right] \right\}$$

$$H_\rho = \frac{I l \sin \phi e^{-(\gamma_0 R' - \gamma_1 z)}}{4\pi(R')^3} \left\{ \frac{R'_1}{R'_i} (d + h) \left[ \left( \frac{R'}{\rho} \right)^2 + \left( \frac{R'_1}{R'_i} \right)^2 (1 + \gamma_0 R'_i) \right] e^{-\gamma_0(R'_1 - R')} \right.$$

$$\left. + h(1 + \gamma_0 R') + \left( \frac{R'}{\rho} \right)^2 (d\gamma_0 R' - h) \right\}$$

$$H_\phi = - \frac{I l \cos \phi e^{-(\gamma_0 R' - \gamma_1 z)}}{4\pi(R')^3} \left\{ \left( \frac{R'}{\rho} \right)^2 \left[ (d + h) \frac{R'_1}{R'_i} e^{-\gamma_0(R'_1 - R')} \right] \right.$$

$$\left. + d(\gamma_0 R' + \gamma_0^2 \rho^2) - h \right\} - 2h(1 + \gamma_0 R')$$

$$H_z = \frac{I l \rho \sin \phi e^{-(\gamma_0 R' - \gamma_1 z)}}{4\pi(R')^3} \left\{ (1 + \gamma_0 R') - \left( \frac{R'}{R'_i} \right)^3 (1 + \gamma_0 R'_i) e^{-\gamma_0(R'_1 - R')} \right\}$$


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Table 6. HMD Air-to-Subsurface Propagation Equations  
 $[(R')^2 = \rho^2 + h^2, (R'_i)^2 = \rho^2 + (d + h)^2]$

$$\begin{aligned}
 E_\rho &= \frac{i\omega\mu_0 IA \cos \phi e^{-(\gamma_0 R' - \gamma_1 z)}}{4\pi\rho^2 R'} \left\{ (d + h) \frac{R'}{R'_i} e^{-\gamma_0 (R'_i - R')} \right. \\
 &\quad \left. - h + d(\gamma_0 R' + \gamma_0^2 \rho^2) \right\} \\
 E_\phi &= \frac{i\omega\mu_0 IA \sin \phi e^{-(\gamma_0 R' - \gamma_1 z)}}{4\pi\rho^2 R'} \left\{ \gamma_0 d R' - h - \frac{\rho^2 h}{(R')^2} (1 + \gamma_0 R') \right. \\
 &\quad \left. + (d + h) \frac{R'}{R'_i} \left[ 1 + \frac{\rho^2}{(R'_i)^2} (1 + \gamma_0 R'_i) \right] e^{-\gamma_0 (R'_i - R')} \right\} \\
 E_z &= \frac{i\omega\mu_0 I A \rho \cos \phi}{2\pi n^2 (R')^3} (1 + \gamma_0 R') e^{-(\gamma_0 R' - \gamma_1 z)} \\
 H_\rho &= \frac{IA \sin \phi e^{-(\gamma_0 R' - \gamma_1 z)}}{4\pi (R')^3} \left\{ \left[ 2 - \frac{3h^2}{(R')^2} \right] \left[ 1 + \gamma_0 R' \right] - 2\gamma_0^2 h^2 - \gamma_0^2 \rho^2 \right. \\
 &\quad \left. + \left[ \left[ 2 - \frac{3(d + h)^2}{(R'_i)^2} \right] \left[ 1 + \gamma_0 R'_i \right] + \gamma_0^2 \rho^2 \right] \left( \frac{R'}{R'_i} \right)^3 e^{-\gamma_0 (R'_i - R')} \right\} \\
 H_\phi &= \frac{IA \cos \phi e^{-(\gamma_0 R' - \gamma_1 z)}}{4\pi (R')^3} \left\{ 1 + \gamma_0 R' + \gamma_0^2 (R')^2 \right. \\
 &\quad \left. + \left( \frac{R'}{R'_i} \right)^3 (1 + \gamma_0 R'_i) e^{-\gamma_0 (R'_i - R')} \right\} \\
 H_z &= \frac{I A \rho \sin \phi e^{-(\gamma_0 R' - \gamma_1 z)}}{4\pi (R')^5} \left\{ (d + h) \left( \frac{R'}{R'_i} \right)^5 \left[ 3 + 3\gamma_0 R'_i \right. \right. \\
 &\quad \left. \left. + \gamma_0^2 (R'_i)^2 \right] e^{-\gamma_0 (R'_i - R')} - h \left[ 3 + 3\gamma_0 R' + \gamma_0^2 (R')^2 \right] \right\}
 \end{aligned}$$

When  $|\gamma_1 R'| \gg 1$ , the HED and HMD air-to-subsurface propagation equations reduce to the nearfield range results presented in table 3.5 of Kraichman.<sup>19</sup>

#### SUBSURFACE-TO-SUBSURFACE PROPAGATION

The HED and HMD image-theory expressions for the subsurface-to-subsurface propagation case ( $h \leq 0, z \leq 0$ ) can be obtained from the air-to-air propagation equations (tables 1 and 2) simply by setting both  $z$  and  $h$  equal to zero and multiplying each expression by  $\exp[\gamma_1(z + h)]$ . (Both  $E_z$  components must also be multiplied by  $1/n^2 = \gamma_0^2/\gamma_1^2$  to satisfy the boundary conditions.) The resulting expressions are presented in tables 7 and 8 and should be valid<sup>18</sup> (for most cases) for  $\rho > 3|z + h|$ ,  $|n^2| > 15$ , and small numerical distances. When  $|\gamma_0 \rho| \ll 1$  and  $z = h = 0$ , these expressions reduce to the quasi-static range surface-to-surface image-theory results.<sup>1,2</sup> When  $|\gamma_1 \rho| \gg 1$ , they reduce to the subsurface-to-subsurface nearfield and farfield range results presented in tables 3.2 and 3.7 of Kraichman.<sup>19</sup>

#### COMPARISON WITH EXACT SOMMERFELD INTEGRATION RESULTS

Mitra et al.<sup>20</sup> have presented some exact Sommerfeld integration results for the HED  ${}_0\Pi_x$  and  $\Pi_z$  vectors (i.e., the correction terms to the perfectly conducting ground solution) for frequencies of 3 to 30 MHz. For their case,  $R_1 = 10$  m,  $\theta = \tan^{-1}[\rho/(z + h)] = 10^\circ$ ,  $\phi = 0^\circ$ , and the quantity  $I_0 = I_2/(i\omega\epsilon_0)$  is normalized to unity.

The image-theory solution of  $\Pi_z$  is given by equation (37), while  ${}_0\Pi_x$  is given by the last two terms of equation (24); that is,

$${}_0\Pi_x \sim \frac{I_0}{4\pi} \left[ \frac{e^{-\gamma_0 R_1}}{R_1} - \frac{e^{-\gamma_0 R_2}}{R_2} \right]. \quad (44)$$

A comparison of the image theory and exact Sommerfeld integration results for the situation where  $\epsilon_r = 40$  and  $\sigma_1 = 1$  S/m is presented in figure 3. For this situation,  $n^2$  varies from  $40 - j150$  at 3 MHz to  $40 - j15$  at 30 MHz. Since  $|n^2| > 15$ ,  $d_{TM} \sim d_{TE} \sim d = 2/\gamma_1$ . Note that the agreement between the two solutions is excellent.

Presented in figure 4 is a comparison of the image-theory and exact Sommerfeld integration results for the case where  $\epsilon_r = 10$  and  $\sigma_1 = 10^{-2}$  S/m. For this case,  $n^2$  varies from  $10 - j6$  at 3 MHz to  $10 - j0.6$  at 30 MHz. Since  $|n^2| < 15$  and  $(z + h) \gg \rho$ , we have replaced  $\gamma_0 d$  by  $1 - \exp(-\gamma_0 d)$  in the image theory  $\Pi_z$  expression (equation (37)). Note that the agreement is excellent (within 1 percent for the  ${}_0\Pi_x$  component and very good (within 5 percent) for the  $\Pi_z$  component).

Table 7. HED Subsurface-to-Subsurface Propagation  
Equations ( $\rho_1^2 = \rho^2 + d^2$ )

$$E_\rho = \frac{I l \cos \phi e^{-[\gamma_0 \rho - \gamma_1(z+h)]}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left\{ 2 + 2\gamma_0 \rho + \gamma_0^2 \rho^2 - \frac{2\rho^2}{d^2} \left[ 1 - \frac{\rho}{\rho_i} e^{-\gamma_0(\rho_i - \rho)} \right] \right\}$$

$$E_\phi = \frac{I l \sin \phi e^{-[\gamma_0 \rho - \gamma_1(z+h)]}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left\{ 1 + \gamma_0 \rho + \frac{2\rho^2}{d^2} \left[ 1 - \frac{\rho}{\rho_i} e^{-\gamma_0(\rho_i - \rho)} \right] \right\}$$

$$E_z = \frac{I l \gamma_1 \cos \phi e^{-[\gamma_0 \rho - \gamma_1(z+h)]}}{2\pi(\sigma_1 + i\omega\epsilon_1)n^2 \rho^2} \left\{ \gamma_0 \rho + \frac{\rho}{\rho_i} e^{-\gamma_0(\rho_i - \rho)} \right\}$$

$$H_\rho = \frac{I l \sin \phi e^{-[\gamma_0 \rho - \gamma_1(z+h)]}}{2\pi\gamma_1 \rho^3} \left\{ \frac{\rho}{\rho_i} \left[ 1 + \frac{\rho^2}{\rho_i^2} (1 + \gamma_0 \rho_i) \right] e^{-\gamma_0(\rho_i - \rho)} + \gamma_0 \rho \right\}$$

$$H_\phi = - \frac{I l \cos \phi e^{-[\gamma_0 \rho - \gamma_1(z+h)]}}{2\pi\gamma_1 \rho^3} \left\{ \frac{\rho}{\rho_i} e^{-\gamma_0(\rho_i - \rho)} + \gamma_0 \rho + \gamma_0^2 \rho^2 \right\}$$

$$H_z = \frac{I l \sin \phi e^{-[\gamma_0 \rho - \gamma_1(z+h)]}}{4\pi\rho^2} \left\{ (1 + \gamma_0 \rho) - \frac{\rho^3}{\rho_i^3} (1 + \gamma_0 \rho_i) e^{-\gamma_0(\rho_i - \rho)} \right\}$$



Table 8. HMD Subsurface-to-Subsurface Propagation  
Equations ( $\rho_i^2 = \rho^2 + d^2$ )

---


$$E_\rho = \frac{IA\gamma_1 \cos \phi e^{-[\gamma_0 \rho - \gamma_1(z+h)]}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left\{ \frac{\rho}{\rho_i} e^{-\gamma_0(\rho_i - \rho)} + \gamma_0 \rho + \gamma_0^2 \rho^2 \right\}$$

$$E_\phi = \frac{IA\gamma_1 \sin \phi e^{-[\gamma_0 \rho - \gamma_1(z+h)]}}{2\pi(\sigma_1 + i\omega\epsilon_1)\rho^3} \left\{ \frac{\rho}{\rho_i} \left[ 1 + \frac{\rho^2}{\rho_i^2} (1 + \gamma_0 \rho_i) \right] e^{-\gamma_0(\rho_i - \rho)} + \gamma_0 \rho \right\}$$

$$E_z = \frac{i\omega\mu_0 IA \cos \phi e^{-[\gamma_0 \rho - \gamma_1(z+h)]}}{2\pi n^2 \rho^2} (1 + \gamma_0 \rho)$$

$$H_\rho = \frac{IA \sin \phi e^{-[\gamma_0 \rho - \gamma_1(z+h)]}}{4\pi \rho^3} \left[ 2(1 + \gamma_0 \rho) - \gamma_0^2 \rho^2 \right. \\ \left. + \left( \frac{\rho}{\rho_i} \right)^3 e^{-\gamma_0(\rho_i - \rho)} \left[ \left[ 2 - \frac{3d^2}{\rho_i^2} \right] [1 + \gamma_0 \rho_i] + \gamma_0^2 \rho_i^2 \right] \right]$$

$$H_\phi = \frac{IA \cos \phi e^{-[\gamma_0 \rho - \gamma_1(z+h)]}}{4\pi \rho^3} \left\{ 1 + \gamma_0 \rho + 2\gamma_0^2 \rho^2 + \left( \frac{\rho}{\rho_i} \right)^3 (1 + \gamma_0 \rho_i) e^{-\gamma_0(\rho_i - \rho)} \right\}$$

$$H_z = \frac{IA \rho \sin \phi e^{-[\gamma_0 \rho_i - \gamma_1(z+h)]}}{2\pi \gamma_1 \rho_i^5} (3 + 3\gamma_0 \rho_i + \gamma_0^2 \rho_i^2)$$


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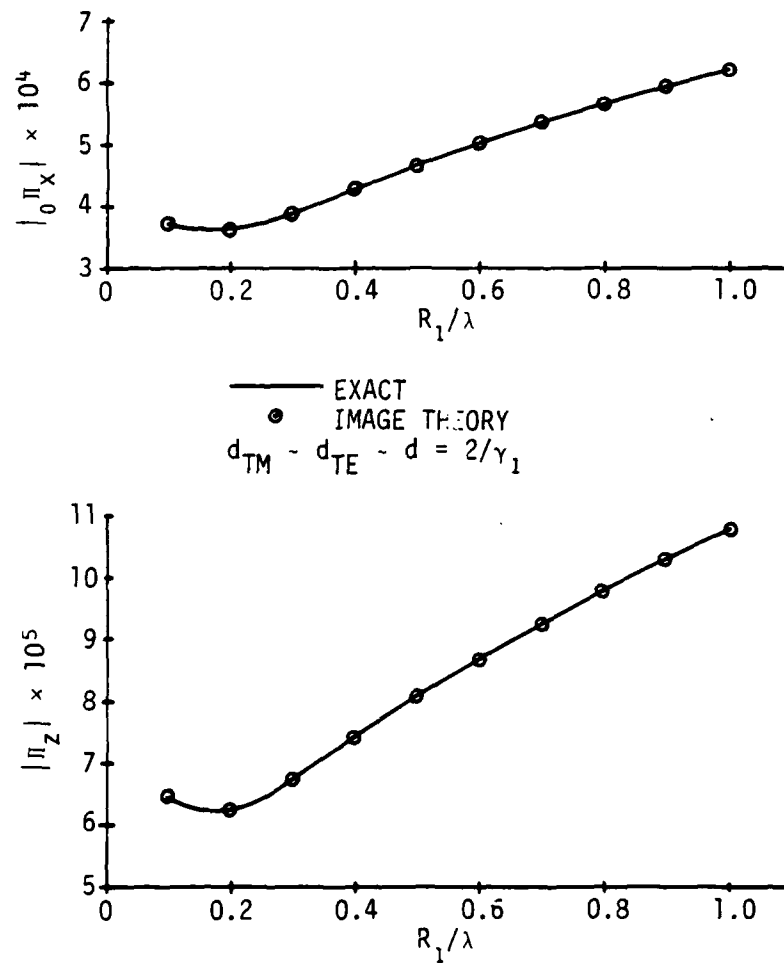


Figure 3. Comparison of Image-Theory and Exact Sommerfeld Integration  
Results for  $\epsilon_r = 40$ ,  $\sigma_1 = 1$  S/m,  $R_1 = 10$  m,  $\theta = 10^\circ$ ,  
 $\phi = 0^\circ$ , and  $f = 3$  to 30 MHz

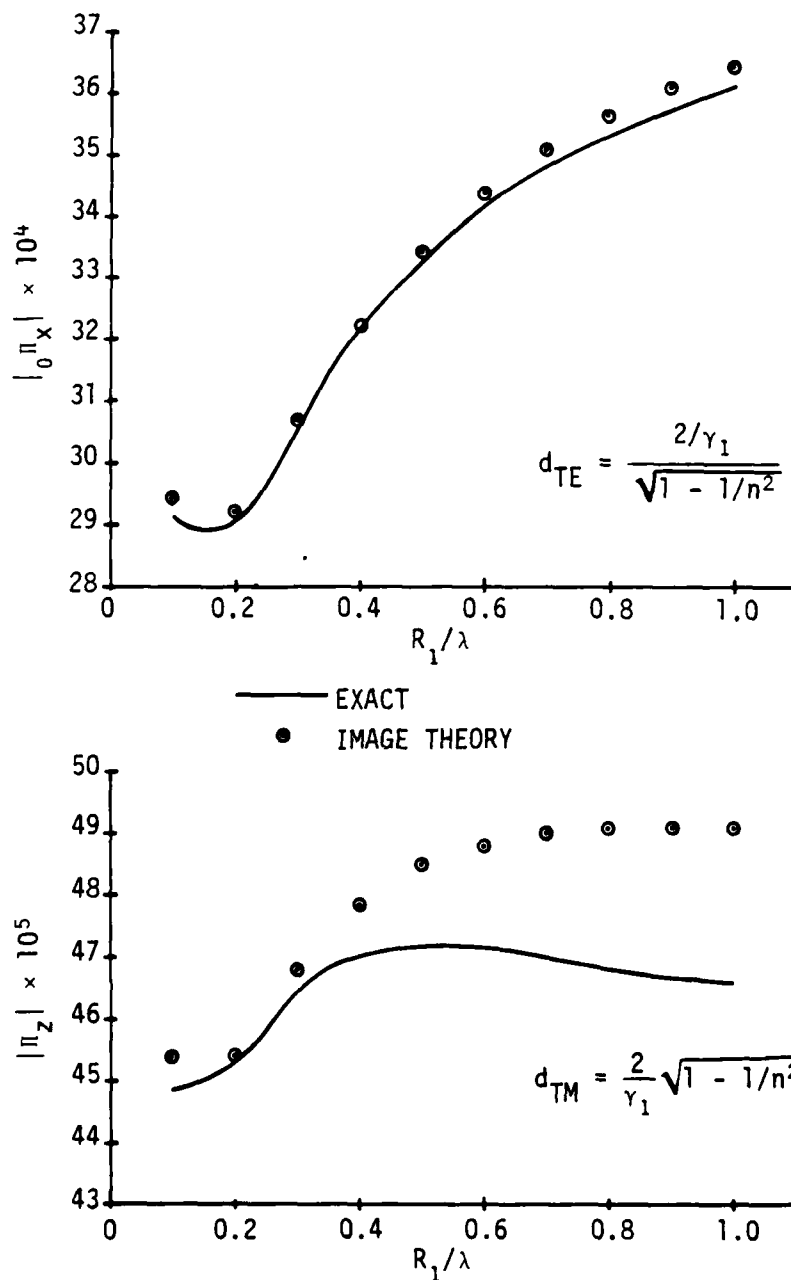


Figure 4. Comparison of Image-Theory and Exact Sommerfeld Integration  
Results for  $\epsilon_r = 10$ ,  $\sigma_1 = 10^{-2}$  S/m,  $R_1 = 10$  m,  $\theta = 10^\circ$ ,  
 $\phi = 0^\circ$ , and  $f = 3$  to 30 MHz

## EXTENSION TO LARGE NUMERICAL DISTANCES

Image theory also can be utilized to determine the fields at large numerical distances (i.e.,  $|p| = |-\gamma_0 R_1 / 2n^2| \gg 1$ ). For the sake of simplicity, we will let  $z = h = 0$  and  $|n^2| \gg 1$ . For this case, equation (18) reduces to

$$\Pi_z = \frac{I \ell \cos \phi}{4\pi i \omega \epsilon_0} \times \frac{\partial}{\partial \rho} \int_0^\infty \left( \frac{2u_0}{u_1 + u_0} \right) \left( \frac{n^2 - 1}{u_1 + n^2 u_0} \right) J_0(\lambda \rho) \frac{\lambda}{u_0} d\lambda. \quad (45)$$

Since  $|\gamma_1 \rho| \gg 1$ ,  $u_1 \sim \gamma_1 = 2/d$ , and

$$\left( \frac{2u_0}{u_1 + u_0} \right) \sim 1 - e^{-u_0 d} - u_0 d. \quad (46)$$

Furthermore, because  $|p| \gg 1$ ,

$$\frac{n^2 - 1}{u_1 + n^2 u_0} \sim \frac{n^2 d}{2} e^{-n^2 d u_0 / 2}. \quad (47)$$

Therefore,

$$\begin{aligned} \Pi_z &\sim \frac{I \ell d \cos \phi}{4\pi i \omega \epsilon_0} \left( \frac{n^2 d}{2} \right) \int_0^\infty e^{-n^2 d u_0 / 2} J_0(\lambda \rho) d\lambda \\ &= - \frac{I \ell \cos \phi}{2\pi \gamma_1 i \omega \epsilon_0} (\gamma_0^2) \left( \frac{n^2 d}{2} \right)^2 \left( \frac{\rho e^{-\gamma_0 D}}{D^3} \right), \end{aligned} \quad (48)$$

where  $D^2 = \rho^2 + (n^2 d / 2)^2$ .

Because  $\rho^2 \gg (n^2 d / 2)^2$ ,

$$\begin{aligned} H_\phi &= -i \omega \epsilon_0 \left( \frac{\partial \Pi_z}{\partial \rho} \right) = \frac{I \ell \cos \phi (\gamma_0^2)}{2\pi \gamma_1 \rho} \left( \frac{n^2}{\gamma_0 \rho} \right) e^{-\gamma_0 \rho} \\ &= - \frac{I \ell \cos \phi (\gamma_0^2) e^{-\gamma_0 \rho}}{2\pi \gamma_1 \rho} \left( - \frac{1}{2\rho} \right), \end{aligned} \quad (49)$$

which is the correct farfield result for large numerical distances.

## CONCLUSIONS

Simple engineering expressions for HED and HMD air-to-air, subsurface-to-air, air-to-subsurface, and subsurface-to-subsurface propagation have been derived by employing finitely conducting earth-image theory techniques. For the air-to-air propagation case, the expressions are valid from the quasi-static to the farfield ranges as long as  $|n^2| > 15$  and the Sommerfeld numerical distance is small. For the subsurface-to-air, air-to-subsurface, and subsurface-to-subsurface cases, the additional restriction that the measurement distance be greater than three times the burial depth of the source and/or receiver must be met. We have also demonstrated that image theory can be utilized to determine the fields at large numerical distances.

We have compared successfully image-theory and exact Sommerfeld integration results for four cases, yielding agreement within 1 percent for three comparisons and within 5 percent for the other.

It should be noted that the two media can be inverted and the air replaced by the earth's crust (of conductivity  $\sigma_2$  and dielectric constant  $\epsilon_2$ ). The same equations (tables 1 through 8) can be utilized as long as  $|\gamma_2 R_1 / (2n_2^2)| \ll 1$  and  $|n_2^2| = |\gamma_1^2 / \gamma_2^2| > 15$  simply by replacing  $i\omega\epsilon_0$  by  $\sigma_2 + i\omega\epsilon_2$ .

The results presented in this report should be particularly useful for sea/air and sea/earth's-crust propagation. They should also be helpful to geophysicists engaged in determining the electrical properties of the earth. The simple, yet accurate, formulas obtained from this theory make it a very strong tool and a promising one for determining the coupling between antennas located above or below the earth's surface.

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